UNCLASSIFIED

AD 274 210

Reproduced
by the

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

274210

OPERATIONAL MATHEMATICS SERIES NO. 8

THE SPECIAL MATHEMATICAL DISCIPLINES OF OPERATIONS RESEARCH

November 1961

DEPARTMENT OF THE ARMY

OFFICE OF THE QUARTERMASTER GENERAL

THE SPECIAL MATHEMATICAL DISCIPLINES OF OPERATIONS RESEARCH

by

John K. Sterrett
and
Herbert Maisel

November 1961

QM OPERATIONAL MATHEMATICS SERIES
REPORT NO. 8

FOREWORD

One of the developments in recent decades has been the use of mathematics in areas that were once considered too qualitative or subjective to permit mathematical treatment. This has had a profound effect on mathematics itself; mathematicians now speak of modern mathematics when referring to the disciplines used. Because of the importance of the use of modern mathematics in Quartermaster Operations Research, a description in non-mathematical terms of the mathematical disciplines used is presented in this report.

One of the authors, Dr. Sterrett, was the first Chief of the Quartermaster Corps' Operational Mathematics Office; the other, Mr. Maisel, has been employed in this Office since 1959. Dr. Sterrett, upon his departure, left a cogent description of the mathematics of Operations Research. Mr. Maisel, utilizing Dr. Sterrett's records, developed this report, the eighth in a series of Operational Mathematics Reports.

B. E. KENDALL
Brigadier General, USA
Acting Deputy The Quartermaster General

TABLE OF CONTENTS

INTRODUCTION

Operations Research is an attitude, an approach, a concept. It is a scientific methodology -- analytical, experimental, quantitative -- which, by assessing the over-all implications of various alternative courses of action in a system under evaluation, provides an improved basis for decision. Its general mode of operation is in fact to attempt to choose a single course of action from a large body of possible actions. It always seeks a "best" solution or a "least" costly approach. Thus there is associated with it the terms "maximum" and "minimum" and the goal of optimizing. These have obvious mathematical implications.

But operations research offers much more than rigid optimum decisions. Certainly in many and perhaps in most applications the problem cannot be formulated so as to permit a really best solution. For example, we may be weighing performance against cost in a situation where it is not possible to decide beforehand on the optimum desirable combination of cost and performance. In such cases the techniques of operations research would provide a <u>set of alternative solutions</u> giving the performance and cost associated with each and in this way provide an improved basis for decision. In effect then the operations research approach has also provided an indication of the relation between cost and performance as an important by-product.

Operations Research employs a team approach of some sort to most of the problems it attempts to solve. In this manner it gains one of its many definitions -- that it is an "interdisciplinary activity". Almost invariably in recent times, one of these team disciplines, mathematics, has always been present to some extent. The trend has been to consider more and more that operations research is an application of quantitative techniques to those areas which formerly were thought to be capable of being studied only through qualitative means.

Operations Research approaches have certain basic characteristics which tend to identify the close inter-relationship of operations research to management engineering, research methodology, and mathematical formulation. These basic characteristics are:

a. Operations Research is concerned with the problems of operations of a system as a whole. No particular components -- human or otherwise -- receives special emphasis. Each is considered only insofar as it contributes to the objectives of the system.

b. Operations Research utilizes the scientific method in that it is analytical, experimental, and quantitative. It seeks specific well-defined goals sometimes using the approach of the experimental scientist to reach them.

c. Operations Research techniques almost invariably involves model building, which is fundamental to the scientific approach. Of course, there are many types of models -- simulation, mathematical, physical.

d. As noted above, Operations Research studies almost invariably involve predictions of the effects of alternate courses of action.

e. Operations Research seeks to discover regularities in apparently unrelated or random activities, leaning heavily on the tools of mathematical statistics to build order from chaos and to permit precise analyses in the presence of appreciable fluctuations.

f. Operations Research is equally interested in improving what already exists and in developing something new. The greatest emphasis in the past has been in the latter; however, it will no doubt gain its greatest rewards from the former.

Since operations research is becoming so dependent upon quantitative techniques, it is now sometimes thought of as a discipline of mathematics. This is not entirely so. It seems better to say that certain very special techniques of mathematics have been developed which are having their greatest utilization in operations research. Some of these are brand new techniques. Others are revisions or modifications of those that have been in use for a long time.

The mathematical techniques are used in both the formulation of problems into a form capable of quantitative consideration as well as in the analytic manipulations which lead to those unusual results currently attributed to the operations research methods. In this report we will discuss what some of these special mathematics disciplines are.

In operations research a pattern of regularity is generally represented as a "model" of some sort. More and more this model which is produced is mathematical in nature or leads quite easily into this category. Certain typical situations, repeatedly met, have inspired the development of typical mathematical formulation methods and their associated analytical procedures. These latter contribute significantly to the manipulations of a mathematical nature encountered in present day operations research. Listed here in a group, and described in further detail on the succeeding pages, are some of the more important mathematical disciplines now in extensive use in operations research.

Linear programming	Stochastic processes	Non-linear programming
Information theory	Monte Carlo techniques	Dynamic programming
Gaming theory	Value theory	Search theory
Decision theory	Model theory	Boolean algebra
Queuing theory	Symbolic logic	

The applications of these techniques have many ramifications. New ones will be needed. It is predicted that the further development of these, and the research needed to produce new types of disciplines as the usage of these gain in momentum, will occupy many mathematical statisticians, analysts and theorists for many years yet to come. This field is not in any sense of the word static at its present state.

Traditional mathematics -- algebra, geometry, calculus -- play an important role in operations research. However, some of the more modern developments in mathematics have gone hand in hand with the development of the relatively new field of operations research. For example, some of the first practical applications of symbolic logic are credited to operations research scientists. Similarly, operations research has taken full advantage of both linear and non-linear algebras in the solution of programming and other problems. In fact, operations research is having considerable impact on the "traditional" approach to mathematics in the colleges and universities of the United States and is influencing mathematical curricula in the direction of inclusion of additional courses, particularly in higher algebra.

Operations Research has also borrowed freely from scientific areas other than mathematics. These other things borrowed have been used essentially in developing its set of concepts and tools, its modes of operation, and its reasoning towards logical conclusions. It will be observed that even these are largely mathematical in nature. Some of those more worthy of special attention are:

The uncertainty principle of physics
Cost analysis of economics
Group concepts of social psychology
Trend analysis of history
Cause and effect views of chemistry
Information flow of biology.

The last section contains a list of references on the topics discussed in this report. Since the style of this report is non-mathematical, it was decided to provide the reader with somewhat more mathematical references so that he may see the subject discussed in its natural language. In a few instances the writings of the individual originating the subject is suggested as a reference.

Linear programming can be thought of as a computational technique for finding the maximum or minimum of a linear function subject to certain linear types of inequalities.

The kind of problem which is suitable for linear programming occurs in most operational and research activities. One of its very early uses had to do with the conditions necessary to achieve optimum diet at the minimum cost. This problem was approached during World War II, when various problems of this type were encountered. During the last ten years the techniques have been greatly improved. The high speed computer has been a major contributing factor in these needed improvements.

An appropriate understanding of the rigorous mathematical foundations of the linear programming techniques requires some knowledge of convex sets and matrix manipulations. However, the actual computational procedure of linear programming is quite simple by comparison, though most laborious.

The real difficulty in applications of linear programming occurs at almost the very beginning of a problem, i.e., in the early stage when the problem to be solved is specified. The isolating of activities and the estimation of their parameters is not always an easy problem and requires considerable individual insight into the problem. Sometimes it is essential to introduce certain "dummy" activities to obtain the necessary linearity, so that considerable experience is required before the technique can be utilized.

There are many important problems which can and should be solved by linear programming. However, it is not always usable in all cases. It is not the one type of solution which serves all purposes and solves all of everyone's problems. Extreme care must be employed in choosing whether or not it should be used at all in many of the problems to which it is directed. Some examples of problems which it has already solved are:

> Determining the optimum combination of inputs into a manufacturing process which will maximize profits. (Optimum Input Problem)
>
> Given that various foodstuffs contain certain vitamins in fixed but different proportions, and given the minimum requirements of

these vitamins, how can this requirement be met in the cheapest way in terms of shipping space or transportation costs. (Optimum Diet Problem)

Determine which of a series of bids on a procurement action are minimum. (Bid Evaluation Problem)

Another major area of application is in determining the optimum allocation, or use, of limited resources to achieve some desired objective.

The resources may be defined to be:

a. The money that is available for use.

b. The organization or individual laboratory capacity.

c. The scientific skills available for a specified task.

The desired objective might be:

a. The lowest cost to achieve a given result.

b. The highest benefit possible from the way in which the resources are used.

The following are identifying characteristics of linear programming:

a. It is assumed that a straight line relationship exists among the variables.

b. The objective of the problem is precisely stated and the restrictions precisely known.

c. The volume of calculations that must be performed is often so extensive that an electronic computer is essential.

Linear programming can be used to determine:

a. Lowest distribution costs from factories to warehouses.

b. Provide better utilization of facilities.

c. Provide better methods of planning.

d. Provide improved cost-outcome relationships.

e. Determine the best combinations of research, development and engineering.

Linear programming is one of the most powerful and broadly used techniques in operations research. The work involved in establishing the necessary equations is usually a task for a well trained individual. The techniques and solutions require inspired mathematical disciplines. Controlled scientific improvisation is not uncommon.

INFORMATION THEORY

Information theory is an analytical process transferred from the electrical communications field to operations research. It presumes to evaluate the effectiveness of information flow within a given system under study. Its most extensive utilization in management science thus far has been its influence in stimulating the examination of organizational structures with the view to improving information or communication flow.

In information theory the word "communication" is used in a very broad sense to include all of the procedures by which a body of knowledge in one environment may be transferred to another. This, of course, can involve other things than oral or written speech. It involves more than the communication of one human being with another. Thus, in automated processes, machines "talk" to machines and an exchange of information is occurring in the absence of humans. A broader theory, encompassing information theory, is Cybernetics -- the science of control and communication in the machine and the animal.

There are three levels of problems, namely:

a. The Technical Problem: How accurately can the symbols of communication be transmitted?

b. The Semantic Problem: How precisely do the transmitted symbols convey the desired meaning?

c. The Effectiveness Problem: How effectively does the received meaning affect conduct in the desired way?

In the first, the mathematical theory is exceedingly general in its scope, fundamental in the problems it treats, and of classic simplicity and power in the results it reaches.

In the second, probability plays a major role in the generation of the meaning of the message, for the choice of a meaning of successive bits (abbreviation for binary digit) from the point of view of the communication system is governed by probabilities; in fact, from probabilities which are not independent, but which, at any stage of the process, depend on preceding choices.

In the third, it is obvious that in using communication to control we depend on how effectively the conveyed meaning influences conduct. Thus the cybernetician is especially concerned with the Semantic and Effectiveness problems.

GAMING THEORY

Gaming is a mathematical theory that provides a framework for studying and evaluating competitive situations. The study of competition among several factions establishes a mathematical model that can be manipulated for the purpose of determining one player's or one group of players' strategy and most likely gain. It is considered to be one of the more useful ways of demonstrating the likely consequence of a certain course of action and is therefore a most useful decision-making tool. It is one of the more rigorous domains of modern mathematics.

The following are identifying characteristics of gaming theory:

a. The theory makes important use of mathematical logic and mathematical analysis.

b. The theory of games evolves, as a solution of each game of strategy, the distribution or distributions of payments to be made by every player as a function of all other individuals' behavior.

c. Some game theory approaches have an optimal strategy. In this there is a sequence of moves such that the player using it will have the safest strategy possible, whatever his opponent does. In this case his position will not deteriorate even if the strategy is found out. This type of game is called a "strictly determined" game. Every move --

and every position resulting from a series of moves -- is out in the open. In this instance a "saddle point" or better called "single safest strategy" is the desired result. It is desired in the sense that it is safest, i.e., assures the player that he will minimize his maximum possible loss. If the player seeks some other objective, the game is not really strictly determined.

d. Other gaming theory processes have no single safest possible strategy. It is this type which fits most military situations. In these it would be especially disastrous if a player's strategy were discovered by his opponent. The player's particular concern is to protect his strategy from discovery. Several "saddle points" are possible in this case.

e. Frequently the player's strategy is itself determined in part by the result of random processes, such as the flip of a coin or the spin of a wheel. We mean here that in order to determine what his next move should be the player may, for example, select between two alternative moves by flipping a coin. This has the advantage of helping to hide his strategy from an observer who can see only his moves and also avoids subjective biases in deciding on the next move where such biases may prove harmful. A strategy including such random procedures is called a "mixed strategy"; one in which all moves are fixed and pre-determined is called a "pure strategy".

Gaming theory was originally developed -- by a mathematician -- with a view toward certain problems of economic theory. The initial reaction of the economists to this work was one of great reserve, but the military scientists were quick to sense its possibilities in their field, and they have pushed its development. However, through linear programming, it is again feeding back into economic theory, and applications of gaming are becoming more widespread in business and industry.

One of the potentials of gaming theory is its use in what is termed strategic, or operational, gaming. Operational and strategic gaming, in addition to having merits of their own, assist in the design of experiments by helping to define boundary conditions and otherwise eliminating unprofitable approaches. A great advantage of operational gaming, apart from the fact that it is generally less expensive than operational experimentation and does lend itself to many repetitions, is the fact that actual decision-makers may be used to make the decisions that become necessary in the course of the play. In this way, managerial experience, judgement, and intuition can be teamed with the technical skill of the operations research scientists who design the game, serve as umpires in the play, and assess and evaluate the results. Thus it is that one can observe that perhaps the greatest contribution of gaming theory so far has been an intangible one: the general orientation given to people who are faced with overly complex problems. Even though these problems are not strictly solvable -- certainly at the moment and probably for the indefinite future -- it helps to have a framework in which to work on them. The concepts of strategy, the distinctions among players, the role of chance, the notion of matrix representations of the payoffs, the concepts of "pure" and "mixed" strategies, and efforts at abstraction of situations give valuable orientation to persons who must think about complicated conflict situations.

DECISION THEORY

Decision theory is that process which provides the basis for selecting one action from a number of alternative courses of action. In other words, it is a study of how to make decisions. A better name for it would probably be Statistical Decision Theory. In order to make decisions one must be able to trace down the consequences of the alternative lines of action. Three basic steps are present:

The outcomes for each action are predicted.

The outcomes are evaluated in terms of some scale of desirability or utility.

A "criterion" for decision, based on the purposes, is then used to make the actual selection.

Characteristics of a decision theory approach are the following:

The problem of choice: Alternative actions and conflicting values.

The problem of prediction: Probabilities versus certainties.

The problem of action: Calculated risk in the face of uncertainty.

The problem of statistical inference: Mathematical models and Sampling techniques.

Statistical decision attempts to deal with the problems of action in the real world, but there are many ways of looking at the real world. In order to attack the problem -- in order even to "state the problem" -- it is necessary to make some assumptions about the real world. This attitude toward the real world taken by Statistical Decision Theory is consistent with the one accepted by modern science. That is, abstract models of the real world are useful in developing scientific theories and are, in much the same way, important in employing statistical decision theory.

Model formulation and manipulation are paramount requirements in this discipline. All types of models might be used. They are Physical (airplane), Abstract (planetarium), Symbolic (analytical mathematical concepts), and Mathematical (statistical and probabilistical).

QUEUING THEORY

Queuing theory involves those processes of operations research normally associated with waiting-time problems. A waiting-time problem arises when either units requiring service or the facilities which are providing service stand idle, i.e., wait. In waiting-line situations, problems arise because of either:

a. Too much demand on the facilities -- in which case we may say either that there is an excess of waiting time or that there are not enough service facilities.

b. Too little demand -- in which case there is either too much idle facility time or too many facilities.

Problems involving waiting-time fall into two different types depending on their structure:

a. The first type of problem involves arrivals which are randomly spaced and/or service time of random duration. This class of problems includes situations requiring either determination of the optimal number of service facilities or the optimum arrival rate (or time of arrival), or both.

b. The second type of waiting-time problem is not concerned with either controlling the times of arrivals or the number of facilities, but rather is concerned with the order or sequence in which service is provided to available units by a series of service points.

The first type is generally classed as the waiting-line problem. The second type is generally referred to as the sequencing problem. Both involve rather complex mathematics in the associated model construction and the evaluation manipulations. Many waiting-line problems have been made simpler by the use of Monte Carlo procedures. The sequencing problem requires combinatory analysis for their solution.

Characteristic of queuing theory problems are the following:

a. Arrivals: The manner in which units arrive and become part of the waiting line.

b. Servicing: The number of service units and the service policy, e.g., amount of service that can be rendered.

c. Queue Discipline: The order in which units are served.

d. Output of the System: The service provided and its duration.

STOCHASTIC PROCESSES

A Stochastic Process may be thought of as a mathematical system that studies a probabilistic structure developed in a sequential pattern. For example, a process is stochastic if it includes random variables whose values depend on a parameter such as "time".

Just as the discovery of the laws of gravity are attributed in legend to Newton's observation of a falling apple, so the first work with stochastic processes goes back to a legendary mathematician observing the perambulations of a saturated drunk. Each of the drunk's steps was supposed to have an equal probability of going in any direction. The mathematician wondered how many steps the drunkard would have to take, on the average, to get a specified distance away from his starting point. This was called the problem of "random walk".

Markov chains, useful in studying the development of atomic reactions, are an outgrowth of this study of the random walk. A Markov chain is merely a random walk generalized to permit a variety of probabilities and directions for "the drunk's next step". A formal and effective mathematical theory has developed on this subject.

In addition to a stochastic process built up from a series of distinct steps, we might have one that develops continuously rather than discretely. Thus in automated oil refineries the rate of flow of oil through a critical channel is a continuous function of time. Stochastic theory is useful in controlling this flow in a rigorous way.

MONTE CARLO TECHNIQUES

Monte Carlo techniques are an especially useful mathematical tool. It is a procedure by which we can obtain approximate evaluations of mathematical expressions which are built up of one or more probability distribution functions. Such types of expressions are quite common in the models used in operations research. It is able to give at least approximate answers to many questions where other mathematical techniques fail. The Monte Carlo method is used to solve problems which depend in some important way upon probability -- problems where physical experimentation is impractical and the development of exact formulas is impossible.

Essentially the Monte Carlo method goes back to probability theory, which was developed from studies of gambling games. But it takes the opposite approach. The mathematicians who originated probability theory derived their equations by abstract consideration of gambling games; the Monte Carlo approach would proceed by actually playing a number of gambling games until a sufficient number have been played to derive accurate conclusions about the underlying probability structure. Clearly, high speed computers are a "must" in most of the applications. In fact, one can think of the Monte Carlo method as a device for solving mathematical problems by having one of the giant computers play gambling games with itself. With the Monte Carlo method high speed computers can answer such questions as these:

a. How should the schedule be changed to accommodate a market changed so as to demand twice as many chairs as tables?

b. How much could the shop produce, and at what cost, if one man should be absent for two days?

c. How much would the total output be increased if one man should increase his work rate 20%?

d. Under a given schedule of work flow, what percentage of the time are the men idle because the work piled up behind a bottleneck machine?

VALUE THEORY

Value theory is a process of assigning numerical significance to the worth of alternative choices. It is a special application of many of the techniques employed in decision theory. To date, the formal approach to value theory itself has been to consider it entirely as a theoretical concept. At the moment it is in a status of fundamental model formulation and experimentation. When it is more thoroughly developed, this technique should be most helpful in assessing the worth of the various conclusions in the decision-making process.

Value theory deals with situations in which quantitative preferences clearly exist, but in which a numerical scale of such preferences is not initially available. To illustrate its use, consider the following problem:

> At any moment, the Navy will usually have many thousands of sets of various electronic equipment, each one of which could be installed on many different ships. It is observed that these ships usually have older models of this electronic equipment already installed. Of the huge number of possible allocations, which one will best serve the needs of the Navy?
>
> The problem would be relatively easy if one had a reliable quantitative measure of the gain in the effectiveness of each ship type when a particular equipment is replaced by a new model. Since allocation cannot stop while awaiting the completion of studies to make such a determination, the decision must be based on qualitative preference stated by those best suited to judge.

Now this is where the value theory needs to be applied. Granted that it is necessary in this case to rely on qualitative human judgement, the basic question is this: At what level should the judgement be applied? Accordingly, the approach has been to find a fundamental level at which qualitative human judgement can be applied and the optimal allocation can be "deduced" from this fundamental judgement and other relevant data by "machine computations". Thus, the naval officers do their guesswork at a fundamental level, as distinguished from guessing the entire allocation.

Value theory should have many applications in any of the fields where intangibles play an important role. Proceeding along the line developed in the theory, it is likely that one can soon find that the problem which was initially qualitative and heavily dependent on human judgement has been converted to a quantitative problem. It will be true that qualitative human judgement is still present, but only in a relatively simple form where its effect can be clearly seen.

One final comment. A pragmatic approach toward economically improving system effectiveness called value analysis has developed over the past few years. Though it uses many of the scientific attitudes of the operations research analyst, it should not be confused with mathematical value theory.

MODEL THEORY

Model theory involves the techniques of setting up the proper model and its use. The model is a representation of the system under study. It is a representation which lends itself to predicting the effect on the system's performance of possible changes in the system. It is important to note that there are several types of models: physical, abstract, mathematical, etc. In operations research, the mathematical model is much the more important. By proper mathematical or logical operations, the model can be used to formulate a solution to the problem at hand.

Manipulation of the mathematical model is essential. It varies from mental arithmetic to electronic computing machines. In a sense this consists of an attempt at a solution in terms of given symbols. Mathematical techniques for deriving a solution or optimizing the system are essentially of two kinds: analytical and numerical. In certain models there are terms or variables which cannot be evaluated exactly.

This is the case where Monte Carlo techniques are applicable. Abstraction is the major advantage of all models since it allows us to concentrate attention on those factors considered important. For comparison and information purposes the following model types are given:

Physical models (such as the scale model of an airplane)

The model may be a scaled down portion or a full sized mock-up of a portion of the real situation being studied. This particular type of abstraction, construction of a physical model, is used in various branches of science, engineering, and industry.

Abstract models (such as is used in a planetarium)

Perhaps the greatest use of this type of model in the scientific world is for instructional purposes. It is an excellent means to demonstrate such an event as an eclipse. Verbal models are also in this general category. Most of us are accustomed to using these verbal models in our thinking processes, and we do it intuitively. Verbal models have played an important role in science, especially in the preliminary exploration of a topic and presentation of results. However, most of the scientific fields have advanced to the next stage using symbolic models of a mathematical nature.

Mathematical models (such as an equation to represent a system and its relationships)

It is by this **last** type of model that one reaches the desired degree of abstraction which is so necessary in many complex operations. The construction of symbolic mathematical models is an important part of the job of the scientist, and the great advances in science are those in which a useful new model is introduced. Certain powerful landmarks have occurred in scientific history: Isaac Newton's gravity model, Einstein's relativity theory, and the quantum theory models.

One of the key steps in the progress of a field of knowledge toward scientific maturity is the fabrication of models which enable successful prediction in that field. A tremendous amount of imagination and insight is needed for the creation of new models but they are only part of the story. The mere creation of models in not enough; the models must survive exacting tests and they must meet the pragmatic criterion -- they must work.

Progress in science is based on a constant interplay between model and data. Sometimes there is a tremendous amount of observational data available but no satisfactory model, so that little progress is made. At other times there are elaborate models but little adequate data. Just as in scientific work, in operational applications the model must be closely married to the problem at hand in order to permit the interplay so necessary to make them really work.

SYMBOLIC LOGIC

Symbolic logic is a special process used in making symbolic models of situations and their analytic manipulations in a very abstract manner. Essentially, it involves sets of symbols which have special meanings and which indicate relationships that enter into considerations of a problem. It is closely allied to the laws of thought on which are founded the mathematical theories which have developed through the years. In fact, attempts to explore the logical foundations of mathematics itself rest on this theory. In this effort, mathematical propositions are represented by symbols and manipulated to draw logical conclusions. Thus it is shown that mathematics is in reality a branch of the more primary discipline -- logic.

Symbolic logic is perhaps best described as a means of expressing verbal propositions and statements of relationships between propositions in a concise and unambiguous form. The manipulation of propositions stated in symbolic form is performed by the methods of Boolean Algebra to be discussed later in this report.

The uses of symbolic logic, like the uses of mathematics, cannot be limited to specific applications in a particular field. Just as the same differential equation can describe the mechanical vibrations of a physical system or the flow of electricity in a circuit, a symbolic sentence can describe the conditions of a contract or the operation of a computer element.

For the most part, past activities in formal symbolic logic have remained on a very high level theoretical plain. As such it has tended to be an academic pursuit without application in the engineering or business worlds. It is only in the past twenty years that it has been used in any very practical manner. It is now used to find ambiguities in the fields of insurance and business contracts. It has also found extensive application in the design of computers and circuits and in the design of information retrieval systems.

One might wonder whether some of the solutions to problems brought about by applications of symbolic logic are merely dressed-up versions of common sense or ordinary engineering procedure, and in reality a solution could be obtained in the same manner by someone who knew nothing about formal logic. The answer to this query lies in the definition of symbolic logic. It is a means of clearly expressing verbal propositions and their relationships in symbolic form. Someone who knows nothing of formal symbolic logic may unknowingly use elementary forms of that method and solve problems. This, however, does not detract from the dignity or usefulness of a formal system of symbolic logic and logical operations. The Egyptian mathematicians knew very little of the methods of a formal algebra, but they managed to solve elementary problems. This certainly does not argue against algebra as a formal method, or its dignity as an element of mathematics.

NON-LINEAR PROGRAMMING

Non-linear programming is analogous to linear programming except that the relations encountered are not assumed to be linear relationships, i.e., are not straight-line formulas. The linear programming methods can sometimes be extended to treat cases of mild non-linearity by the method of "small perturbations" -- that is, by studying departures from a known situation which are sufficiently small to permit them to be approximately represented by straight-line formulas. However, real situations often do not obey relationships which are well represented by straight-line formulas. Thus, this new theory is needed. The study of this theory is at the frontiers of modern mathematical research.

DYNAMIC PROGRAMMING

Dynamic programming is another technique which has been developed for use in another situation which is not covered by linear programming. In linear programming the assumption is made that all situations to which it is applied are static. Again, however, real situations exist where this assumption of static conditions is not always a satisfactory approximation. In such cases, there is need for a method which permits changing conditions to occur, and dynamic programming is now being developed for this purpose.

SEARCH THEORY

Search theory is an operations research technique developed for use in the study of ways to minimize the effort required to locate an object. The theory arose originally chiefly from the military problem of locating enemy submarines with limited detection resources. At present it is becoming of importance in other areas, such as in specialized marketing situations.

BOOLEAN ALGEBRA

Boolean Algebra is an algebra of sets or classes of elements which have certain assigned properties depending on the particular problem under consideration. The translation of ordinary language or other communication medium to symbolic form requires an interpretation of the applicable operators and the definition of the symbols. Its greatest use is in the field of high-speed computer logic. It is a form of modern algebra which permits of finite mathematics treatments, which is the case in digital computer activities. It lends itself directly to the "bit" characteristics of the two-digit number system which is the basis of these computer systems.

Boolean algebra is very closely associated with symbolic logic and with sentential calculus. It is of particular use in determining the truth or falsity of an entire statement when the constituent elements take on all possible combinations of truth and falsity.

The abstract laws of Boolean algebra bear a close resemblance to the elementary algebra laws with which most of us are already familiar. For this reason, a set, its subsets and the laws of combination of subsets are considered an algebraic system. It is called Boolean algebra after the British mathematician George Boole who was the first person to study these sets and subsets from an algebraic point of view. We can study any of these systems from either the algebraic or the logical point of view and therein lies its greatest potential, since the theory may be developed using more powerful algebraic manipulations and the result applied to the corresponding problem in logic.

REFERENCES

SUBJECT	AUTHOR(S)	TITLE
Linear Programming	Gass, S. I.	Linear Programming: Methods & Applications
	Vadja, S.	Theory of Games and Linear Programming
Information Theory	Weiner, N.	Cybernetics
	Shannon and Weaver	The Mathematical Theory of Communication
Gaming Theory	Von Neumann and Morgenstern	Theory of Games and Economic Behavior
	Luce and Raiffa	Games and Decisions
Decision Theory	Wald, A.	Statistical Decision Functions
	Blackwell and Girshick	Theory of Games and Statistical Decisions
	Chernoff and Moses	Elementary Decision Theory
Queuing Theory	Brochmeyer, Halstrom and Jensen	The Life and Works of A. K. Erlang, an article in Transactions of the Danish Academy of Technical Sciences, Vol. 2, 1948.
	Saaty, T. L.	Mathematical Methods of Operations Research
	Morse	Queues, Inventories and Maintenance
Stochastic Processes	Feller	An Introduction to Probability Theory and its Applications
	Bartlett	An Introduction to Stochastic Processes
Monte Carlo Techniques	Edited by Herbert A. Meyer	Symposium on Monte Carlo Methods
Non-Linear Programming	Arrow, Hurwicz and Uzawa	Studies in Linear and Non-Linear Programming
	Kuhn and Tucker	Non-Linear Programming, an article in the Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability

REFERENCES

SUBJECT	AUTHOR(S)	TITLE
Dynamic Programming	Bellman, R.	Dynamic Programming
Search Theory	Morse and Kimball	Methods of Operations Research
	Koopman, B. O.	The Theory of Search, an article in Operations Research, Vol. 4, 1956 Nos. 3 and 5
Boolean Algebra & Symbolic Logic	Boole, G.	An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities
	Quine, W. V. O.	Methods of Logic
	Rosser, J. B.	Logic for Mathematicians

DISTRIBUTION

# Copies	
1 Each	All Division/Office Chiefs Office of The Quartermaster General Washington 25, D. C.
1	Colonel G. E. Larsen DCSLOG Washington 25, D. C.
5	Defense General Supply Center Richmond 12, Virginia ATTN: Directorate of Supply Directorate of Quality Control
5	Defense Subsistence Supply Center 226 W. Jackson Blvd. Chicago 6, Illinois ATTN: Mr. Eustice
5	Defense Clothing & Textile Supply Center 2800 South 20th Street Philadelphia 45, Pennsylvania
1	Defense Supply Agency Washington 25, D. C.
1	Colonel C. B. Hazeltine, Jr. Army Research Office Arlington Hall Station Arlington 12, Virginia
1	Lt. Colonel J. F. Wright, Jr. U. S. Army Transportation Research Command Fort Eustis, Virginia
1	Lt. Colonel Anthony W. Urbine Office of The Surgeon General Washington 25, D. C.
1	Colonel Alfred W. DeQuoy Chief, U.S. Army Strategy and Tactics Analysis Group 4815 Rugby Avenue Bethesda 14, Maryland

# Copies	DISTRIBUTION
1	Chief, Operations Research Division Army Research Office, OC/R&D Arlington Hall Station Arlington 12, Virginia
10	Commander Armed Services Technical Information Agency Arlington Hall Station Arlington 12, Virginia
2	Librarian QM Research & Engineering Command Technical Library Natick, Massachusetts
1	The Army Library Rm 1A 518 The Pentagon
1	QM Historian Office of The Quartermaster General Washington 25, D. C.
1	Mr. James Parry Office, Chief of Engineers, Code ENGMS-L Building T-7, Room 2124 Gravelly Point, Va.
1	Mr. A. Sprintzin Defense Supply Agency Building T-7, Room 2102 Gravelly Point, Va.
10	Commanding General QM Research & Engineering Command, US Army Natick, Massachusetts
1	Dr. John K. Sterrett 1311 Iowa Avenue Colorado Springs, Colorado

UNCLASSIFIED

UNCLASSIFIED